Name:

## GCSE (1-9)

## Iteration

## Instructions

- Use black ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
- there may be more space than you need.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out.


## Information

- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1. The equation $x^{3}+7 x-2=55$ has a solution between 3 and 4.

Use trial and improvement to find this solution. Give your answer to 1 decimal place.
2. Use trial and improvement to solve $x^{3}-x^{2}=85$

Give your answer to 1 decimal place.
3. Use trial and improvement to solve $x^{3}+5 x=70$ Give your answer to 1 decimal place.
4. An approximate solution to an equation is found using this iterative process:

$$
x_{n+1}=\sqrt{\left(x_{n}\right)+10} \text { and } x_{1}=3
$$

a) Work out the values of $x_{2}$ and $x_{3}$
b) Work out the solution to 3 decimal places
5. An approximate solution to an equation is found using this iterative process:

$$
x_{n+1}=\frac{\left(x_{n}\right)^{3}-3}{8} \text { and } x_{1}=-1
$$

a) Work out the values of $x_{2}$ and $x_{3}$
b) Work out the solution to 6 decimal places
6. A sequence is defined by the term-to-term rule:

$$
U_{n+1}=U_{n}^{2}-8 \mathrm{U}_{n}+17
$$

a) Given that $U_{1}=4$, find $U_{2}$ and $U_{3}$
b) Given instead that $U_{1}=2$, find $U_{2}, U_{3}$ and $U_{100}$
7.(a) Show that the equation $x^{3}+4 x=1$ has a solution between $x=0$ and $x=1$

$$
(2)
$$

(b) Show that the equation $x^{3}+4 x=1$ can be rearranged to give $x=\frac{1}{4}-\frac{x^{3}}{4}$
(c) Starting with $x_{0}=0$, use the iteration formula $x_{n+1}=\frac{1}{4}-\frac{x_{n}^{3}}{4}$ twice, to find an estimate to the solution of $x^{3}+4 x=1$

